

# Anomalies and WZW-term of two-flavour QCD

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## Abstract

The  $U(2)_R \times U(2)_L$  symmetry of QCD with two massless flavours is subject to anomalies which affect correlation functions involving the singlet currents  $A_\mu^0$  or  $V_\mu^0$ . These are relevant for  $\pi\gamma$ -interactions, because – for two flavours – the electromagnetic current contains a singlet piece. We give the effective Lagrangian required for the corresponding low energy analysis to next-to-leading order, without invoking an expansion in the mass of the strange quark. In particular, the Wess-Zumino-Witten term that accounts for the two-flavour anomalies within the effective theory is written down in closed form.

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# 1 Introduction

It is well known that chiral transformations are, in general, afflicted with anomalies [1,2,3]. In quantum chromodynamics, the most prominent example of an anomalous process is the decay  $\pi^0 \rightarrow \gamma\gamma$ . At low energies, the properties of the pions may be investigated by means of chiral perturbation theory [4,5]. In the effective theory, the anomalies are accounted for by the Wess-Zumino-Witten term [3,6,7,8,9]. In connection with the decay  $\pi^0 \rightarrow \gamma\gamma$ , it is therefore remarkable to note that the effective theory as constructed in ref. [5] involves no WZW-term.

The paradox is, however, easily resolved. It is true in general that a gauge theory is anomaly free if the generators  $t^i$  of the symmetry group satisfy the condition [10]

$$\text{Tr}(t^i \{t^j, t^k\}) = 0, \quad \forall \quad i, j, k.$$

A well known example of such a group is  $\text{SU}(2)$  – the above equation obviously holds if the  $t^i$  are identified with the Pauli matrices. Accordingly, in QCD, the currents of  $\text{SU}(2)_R \times \text{SU}(2)_L$  are anomaly free. In ref. [5], the investigation covered only these – as a consequence a WZW-term did not appear. The point is that this investigation does not cover the electromagnetic interaction: In the case of two quark flavours, the quark charge matrix

$$Q = \text{diag}\{2/3, -1/3\} \tag{1}$$

does not represent a generator of the group  $\text{SU}(2)_R \times \text{SU}(2)_L$  because its trace is different from zero. In order to analyze the Ward identities for Green functions that contain the electromagnetic current, we need to extend the symmetry group to  $\text{SU}(2)_R \times \text{SU}(2)_L \times \text{U}(1)_V$ . In this case, the set of group generators includes the unit matrix and thus the anomalies fail to vanish.

In the present paper, we analyze the anomalies of the full chiral group  $\text{U}(2)_R \times \text{U}(2)_L$ , an investigation which covers the electromagnetic interaction as well as the anomalies of the singlet axial current [11,12]. In section 2 (appendix A) we derive the Ward identities of two-flavour QCD and discuss in particular the anomalous contributions. In the remainder of the paper, we translate the properties of the underlying theory to the effective language. In section 3 we give the leading and next-to-leading order contributions to the nonanomalous part of the chiral Lagrangian. The Wess-Zumino-Witten Lagrangian which accounts for the anomalies in the context of the effective theory is considered in section 4 (appendix B). For two flavours, this term is substantially simpler than the general expression and may be written down in closed form. Our findings are illustrated by means of two examples of three-point functions in section 5.

We point out that although, to our best knowledge, the explicit expression for the WZW-term derived in this paper represents a new result, some of the processes it describes have been studied earlier: The first low energy theorems for anomalous pion interactions were derived prior to the effective Lagrangian formulation [13]. Furthermore, there exist a number of publications investigating the reactions  $\pi^0 \rightarrow \gamma\gamma$  [14] or  $\gamma(\gamma) \rightarrow 3\pi$  [15] in the context of chiral perturbation theory. In these, use was made of the fact that the effective vertices that describe the anomalous interactions of the pions with photons are also contained in the well known expression for the WZW-term for  $SU(3)_R \times SU(3)_L$ . This term itself has been the subject of detailed investigations [16]. Its applications range from the anomalous decays such as  $\eta \rightarrow \pi\pi\gamma\gamma$  [17] to weak interaction kaon physics, where anomalous contributions are the rule rather than the exception [18].

## 2 Ward identities

We consider quantum chromodynamics with two light flavours in the presence of external vector, axial vector, scalar and pseudoscalar fields [5],

$$\begin{aligned} \mathcal{L}_{\text{QCD}} &= -\frac{1}{2g^2} \text{tr}_c G_{\mu\nu} G^{\mu\nu} - \theta \omega + \bar{q} D q + \dots \\ D &= \gamma^\mu (i\partial_\mu + G_\mu + v_\mu + \gamma_5 a_\mu) - s + i\gamma_5 p, \end{aligned} \quad (2)$$

where the coupling constant  $g$  is absorbed in the gluon field  $G_\mu(x)$ . The external fields  $v_\mu(x)$ ,  $a_\mu(x)$ ,  $s(x)$ ,  $p(x)$  represent hermitean, colour neutral  $2 \times 2$  matrices in flavour space. The mass matrix of the two light quarks is contained in the scalar external field  $s(x)$ . The vacuum angle  $\theta(x)$  represents the variable conjugate to the operator  $\omega(x)$ ,

$$\omega = \frac{1}{16\pi^2} \text{tr}_c G_{\mu\nu} \tilde{G}^{\mu\nu}.$$

In Euclidean space, the integral  $\nu = \int dx \omega$  is the winding number of the gluon field, so that  $\omega(x)$  may be viewed as the winding number density. The ellipsis in eq. (2) stands for terms containing the heavy quark fields – what matters in the present context is merely that these are invariant under the subgroup of chiral transformations acting in the space of the light flavours,

$$q(x) \rightarrow \{V_R(x)^{\frac{1}{2}}(1 + \gamma_5) + V_L(x)^{\frac{1}{2}}(1 - \gamma_5)\}q(x) \quad , \quad q = \begin{pmatrix} u \\ d \end{pmatrix}, \quad (3)$$

with  $V_R(x), V_L(x) \in U(2)$ .

The Green functions of the quark currents and the winding number density are generated by the effective action,

$$e^{iS_{eff}\{v,a,s,p,\theta\}} = \langle 0 \text{ out} | 0 \text{ in} \rangle_{v,a,s,p,\theta} .$$

The Ward identities reflect the transformation properties of the effective action under infinitesimal chiral rotations. Formally, the theory is invariant under the transformation in eq. (3), provided one transforms the external fields according to

$$\begin{aligned} v'_\mu + a'_\mu &= V_R(v_\mu + a_\mu)V_R^\dagger + iV_R\partial_\mu V_R^\dagger , \\ v'_\mu - a'_\mu &= V_L(v_\mu - a_\mu)V_L^\dagger + iV_L\partial_\mu V_L^\dagger , \\ s' + ip' &= V_R(s + ip)V_L^\dagger . \end{aligned}$$

It is well known that the quark loops spoil this formal symmetry: The determinant of the Dirac operator,  $\det D$ , is not invariant under the transformation specified in eq. (3). The determinant requires renormalization – there are anomalies because a regularization that preserves chiral symmetry does not exist.

The change in the Dirac determinant generated by an infinitesimal chiral transformation

$$V_R(x) = 1 + i\alpha(x) + i\beta(x) , \quad V_L(x) = 1 + i\alpha(x) - i\beta(x) ,$$

may be given explicitly. The modulus of the determinant is invariant while its phase picks up two distinct contributions. The first is proportional to the winding number density  $\omega$ . It may be absorbed by the transformation

$$\theta \rightarrow \theta - 2\langle\beta\rangle \tag{4}$$

of the vacuum angle (as usual,  $\langle \dots \rangle$  stands for the trace). This part of the change gives rise to the  $U(1)_A$ -anomaly and is independent of the presence of external fields. In contradistinction, the remaining contribution exclusively involves the external fields and may thus be pulled outside the path integral, so that an explicit expression for the change in the effective action can be given. The relevant formula, valid for an arbitrary number of flavours, was first given by Bardeen [2]. We specialize this result to  $N_f = 2$ , where the explicit expression is considerably simpler: With suitably chosen conventions, the anomalies of two-flavour QCD boil down to<sup>1</sup>

$$\delta S_{eff}\{v, a, s, p, \theta\} = -\frac{N_c}{16\pi^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} \langle \hat{\beta}(\hat{v}_{\mu\nu} + i[\hat{a}_\mu, \hat{a}_\nu]) \rangle \langle v_{\rho\sigma} \rangle , \tag{5}$$

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<sup>1</sup>The sign of  $\delta S_{eff}$  is convention dependent; we use the metric  $+- - -$ , set  $\epsilon_{0123} = +1$  and identify  $\gamma_5$  with  $\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$

with  $v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu]$  and the notation

$$\hat{\beta} = \beta - \frac{1}{2}\langle\beta\rangle, \quad \hat{v}_{\mu\nu} = v_{\mu\nu} - \frac{1}{2}\langle v_{\mu\nu}\rangle, \quad \hat{a}_\mu = a_\mu - \frac{1}{2}\langle a_\mu\rangle,$$

for the traceless parts of the  $2 \times 2$  matrices  $\beta$ ,  $v_{\mu\nu}$  and  $a_\mu$ .  $N_c$  denotes the number of colours. A derivation of this result on the basis of Bardeen's formula [2] may be found in appendix A. We add a few remarks:

1. The expression in eq. (5) is independent of singlet axial vector field  $\langle a_\mu \rangle$ . Complications due to the anomalous dimension of the singlet axial current are thus avoided by the conventions chosen, see appendix A.

2. The transformation law in eq. (5) states that the effective action is invariant under chiral  $U(1)_A$ -transformations, where  $\beta \propto \mathbf{1}$ . In view of eq. (4), the anomalies of the correlation functions formed with the singlet axial current are thus accounted for collectively by the operator identity ( $m_q = 0$ )

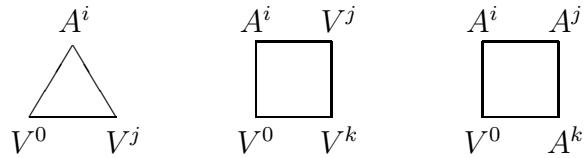
$$\partial^\mu A_\mu^0 = 2\omega, \quad A_\mu^0 = \frac{1}{2}\bar{q}\gamma_\mu\gamma_5 q.$$

3. The remaining anomalies concern correlation functions formed with the currents

$$A_\mu^i = \frac{1}{2}\bar{q}\gamma_\mu\gamma_5\tau^i q, \quad V_\mu^i = \frac{1}{2}\bar{q}\gamma_\mu\tau^i q, \quad V_\mu^0 = \frac{1}{2}\bar{q}\gamma_\mu q,$$

where the  $\tau^i$  denote the Pauli matrices. As eq. (5) shows, the vector currents are anomaly free while for the isovector axial currents  $A_\mu^i$  anomalies occur in the three types of triangle and box diagrams shown in the figure:

Figure 1: Anomalous loop diagrams in two-flavour QCD



Note that, for two flavours, there are no anomalies in pentagon diagrams.

4. The expression for the change in the effective action reduces to zero if the singlet vector field vanishes,  $\langle v_\mu \rangle = 0$ , in accordance with the statement that Green functions built exclusively with the currents of  $SU(2)_R \times SU(2)_L$  are anomaly free. Stated in terms of physical degrees of freedom: There are no anomalies in two-flavour QCD unless at least one of the neutral gauge bosons,  $\gamma$  or  $Z$ , is involved, as in the reactions  $\pi^0 \rightarrow \gamma\gamma$ ,  $\pi^- \rightarrow e^-\bar{\nu}_e\gamma$ ,  $\gamma\pi^0 \rightarrow \pi^+\pi^-$ .

5. As shown by Wess and Zumino [3], the fact that the Ward identities follow from the variation of a single functional imposes nontrivial constraints on the structure of the anomalies. The expression (5) obeys the relevant consistency conditions.

### 3 Low energy expansion

In view of the  $U(1)_A$ -anomaly, the symmetry group of the massless theory is  $SU(2)_R \times SU(2)_L \times U(1)_V$ . We consider the standard scenario and assume that this symmetry is spontaneously broken, the ground state being invariant only under the subgroup  $U(2)_V$ . The low energy properties of the theory are then governed by the pions, the three Goldstone degrees of freedom associated with this breakdown [4,5,6]. It is convenient to collect these fields in a matrix  $U(x) \in U(2)$  that transforms according to the representation

$$U'(x) = V_R(x)U(x)V_L^\dagger(x) . \quad (6)$$

The standard constraint  $\det U = 1$  is not consistent with this transformation law, because the phase of the determinant is not preserved. We replace it by the condition [6]

$$\det U(x) = e^{-i\theta(x)} , \quad (7)$$

which is in accordance with the transformation properties of the vacuum angle (4).

The form of the effective Lagrangian is determined by the symmetry properties of the underlying theory [4,5,6,19]: The low energy expansion of the effective action of QCD is reproduced if the effective Lagrangian consists of (a) the Wess-Zumino-Witten term that reproduces the correct anomalies (see section 4) and (b) the most general expression consistent with chiral symmetry that can be built with the fields  $U$ ,  $v_\mu$ ,  $a_\mu$ ,  $s$ ,  $p$  and  $\theta$  and their derivatives. The terms in the effective Lagrangian are ordered according to their low energy dimension,

$$\mathcal{L}_{eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \dots$$

where the first term is of order  $p^2$ , the second of order  $p^4$ , etc<sup>2</sup>. In view of the  $\epsilon$ -tensor, the WZW-term only shows up at order  $p^4$ . The Lagrangian of order  $p^2$  is thus gauge invariant,

$$\mathcal{L}^{(2)} = \frac{1}{4}F^2 \langle D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U \rangle + \frac{1}{8}h_0 D_\mu \theta D^\mu \theta , \quad (8)$$

with  $\chi \equiv 2B(s + ip)$  and the covariant derivatives

$$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu + \frac{i}{2} D_\mu \theta U , \quad D_\mu \theta = \partial_\mu \theta + 2 \langle a_\mu \rangle , \quad (9)$$

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<sup>2</sup>We use the standard bookkeeping as introduced in [5,6]

where  $r_\mu = v_\mu + a_\mu$ ,  $l_\mu = v_\mu - a_\mu$ . The covariant derivative  $D_\mu U$  transforms in the same manner as  $U$ ,  $D_\mu \theta$  is gauge invariant. Note that we have defined the covariant derivative of  $U$  in such a way that the trace  $\langle U^\dagger D_\mu U \rangle$  vanishes.

At order  $p^4$ , the effective Lagrangian receives two categories of contributions, one of which involves the  $\epsilon$ -tensor while the other does not. Accordingly we decompose the  $p^4$ -Lagrangian in the natural parity part  $\mathcal{L}_g^{(4)}$  (which does not involve the  $\epsilon$ -tensor) and the unnatural parity part  $\mathcal{L}_\epsilon^{(4)}$ ,

$$\mathcal{L}^{(4)} = \mathcal{L}_g^{(4)} + \mathcal{L}_\epsilon^{(4)} .$$

The natural parity part represents a gauge invariant expression. Using the equations of motion associated with the Lagrangian in eq. (8), the most general expression consistent with chiral symmetry,  $C$  and  $P$  reads

$$\begin{aligned} \mathcal{L}_g^{(4)} = & \ell_1 \frac{1}{4} \langle D_\mu U^\dagger D^\mu U \rangle^2 + \ell_2 \frac{1}{4} \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle \\ & + \ell_3 \frac{1}{16} \langle U^\dagger \chi + \chi^\dagger U \rangle^2 + \ell_4 \frac{1}{4} \langle D_\mu U^\dagger D^\mu \chi + D_\mu \chi^\dagger D^\mu U \rangle \\ & + \ell_5 \langle U^\dagger \hat{R}_{\mu\nu} U \hat{L}^{\mu\nu} \rangle + \ell_6 \frac{i}{2} \langle \hat{R}_{\mu\nu} D^\mu U D^\nu U^\dagger + \hat{L}_{\mu\nu} D^\mu U^\dagger D^\nu U \rangle \\ & - \ell_7 \frac{1}{16} \langle U^\dagger \chi - \chi^\dagger U \rangle^2 + \ell_8 D_\mu \theta D^\mu \theta \langle D_\nu U^\dagger D^\nu U \rangle \\ & + \ell_9 D_\mu \theta D_\nu \theta \langle D^\mu U^\dagger D^\nu U \rangle + \ell_{10} D_\mu \theta D^\mu \theta \langle U^\dagger \chi + \chi^\dagger U \rangle \\ & - \ell_{11} i D_\mu \theta \langle D^\mu U^\dagger \chi - D^\mu U \chi^\dagger \rangle + \ell_{12} i \partial_\mu D^\mu \theta \langle U^\dagger \chi - \chi^\dagger U \rangle \\ & + \frac{1}{4} (h_1 + h_3) \langle \chi^\dagger \chi \rangle + \frac{1}{4} (h_1 - h_3) (\det \chi^\dagger e^{-i\theta} + \det \chi e^{i\theta}) \\ & - \frac{1}{2} (\ell_5 + 4h_2) \langle \hat{R}_{\mu\nu} \hat{R}^{\mu\nu} + \hat{L}_{\mu\nu} \hat{L}^{\mu\nu} \rangle + h_4 \frac{1}{4} \langle R_{\mu\nu} + L_{\mu\nu} \rangle \langle R^{\mu\nu} + L^{\mu\nu} \rangle \\ & + h_5 \frac{1}{4} \langle R_{\mu\nu} - L_{\mu\nu} \rangle \langle R^{\mu\nu} - L^{\mu\nu} \rangle + h_6 (D_\mu \theta D^\mu \theta)^2 + h_7 (\partial_\mu D^\mu \theta)^2 , \end{aligned} \quad (10)$$

with the right- and lefthanded field strengths

$$\begin{aligned} R_{\mu\nu} &= \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu] , & \hat{R}_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2} \langle R_{\mu\nu} \rangle , \\ L_{\mu\nu} &= \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu] , & \hat{L}_{\mu\nu} &= L_{\mu\nu} - \frac{1}{2} \langle L_{\mu\nu} \rangle . \end{aligned}$$

The vertices proportional to  $\ell_1, \dots, \ell_7$  and  $h_1, h_2, h_3$  are those relevant for the low energy expansion of the Green functions formed with the  $SU(2)_R \times SU(2)_L$  currents. This part of the effective Lagrangian was given by Gasser and Leutwyler [5] and we have taken over their notation<sup>3</sup>. The additional terms proportional to  $\ell_8, \dots, \ell_{12}$  and  $h_4, \dots, h_7$  are needed for the study of correlation functions involving the operators  $A_\mu^0$ ,  $\omega$  or  $V_\mu^0$ .

The unnatural parity part involves a single free parameter at order  $p^4$ ,

$$\mathcal{L}_\epsilon^{(4)} = \mathcal{L}_{\text{WZW}} + i \tilde{\ell}_1 \epsilon^{\mu\nu\rho\sigma} D_\mu \theta \langle \hat{R}_{\nu\rho} D_\sigma U U^\dagger - \hat{L}_{\nu\rho} U^\dagger D_\sigma U \rangle . \quad (11)$$

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<sup>3</sup>In ref. [5], the effective field is characterized by a four component vector  $U^\Lambda$ . The relation between the two representations reads:  $U^0 = \frac{1}{4} \langle U^\dagger + U \rangle$ ,  $U^k = \frac{i}{4} \langle \tau^k (U^\dagger - U) \rangle$

In the absence of singlet fields this part of the Lagrangian vanishes altogether. The explicit expression for the Wess-Zumino-Witten term  $\mathcal{L}_{\text{wzw}}$  will be given in the following section.

In appendix B of ref. [12], the extensions required by the presence of the singlet fields  $\langle v_\mu \rangle$ ,  $\langle a_\mu \rangle$  and  $\theta$  were discussed in the framework of the effective theory with three light flavours. We briefly mention two observations made there that also apply in the present case:

1. It is well known that the dimension of the singlet axial current,  $A_\mu^0$ , is anomalous [20]. The implications of this fact for the effective theory were worked out in refs. [12,21]: In the effective Lagrangian, the field  $\langle a_\mu \rangle$  as well as some of the effective coupling constants depend on the running scale of QCD. In the present case, the only coupling constants that do depend on the scale are those which multiply terms that involve the covariant derivative  $D_\mu \theta$  or the field strength of the singlet axial current,  $\langle R_{\mu\nu} - L_{\mu\nu} \rangle$ . Under a change of scale these undergo multiplicative renormalization, while all other fields in the effective Lagrangian stay put – in particular, the definition of the covariant derivative  $D_\mu U$  in eq. (9) was chosen on this purpose. The renormalization group invariance of the effective Lagrangian is ensured by the requirement that the effective coupling constants transform contragrediently to the terms they multiply. In particular, the coupling constants  $F, \ell_1, \dots, \ell_7$  of ref. [5] do not depend on the running scale of QCD. The scale dependence of the coupling constants  $\ell_{11}$  and  $\ell_{12}$ , for instance, reflects the fact that the matrix element  $\langle 0 | A_\mu^0 | \pi^0 \rangle$  does not represent a measurable quantity.

2. The second remark concerns the renormalization within the effective theory – note that this issue is completely unrelated to the one discussed above. The one loop graphs associated with the Lagrangian given in eq. (8) require renormalization. In ref. [5], the relevant counterterms have been worked out in the absence of the singlet fields  $\langle v_\mu \rangle$ ,  $a_\mu$  and  $\theta$ . It was shown that the infinities may be absorbed in the coupling constants  $\ell_1, \dots, \ell_6$  and  $h_1, h_2$ . As discussed in ref. [12], this situation is not altered by the presence of the singlet fields: None of the coupling constants  $\ell_8, \dots, \ell_{12}$ ,  $h_4, \dots, h_7$  or  $\tilde{\ell}_1$  is needed to renormalize the one loop graphs of the effective theory.

## 4 Wess-Zumino-Witten term

So far, we have discussed the gauge invariant contributions to the effective Lagrangian. In view of the fact that the effective theory does not involve fermionic degrees of freedom, the effective action generated by a gauge invariant Lagrangian represents a gauge invariant object – the anomalies of the underlying theory are not accounted for. Even before effective La-



grangians were studied systematically, Wess and Zumino [3] pointed out how the anomalies can be accounted for in this framework: In addition to the gauge invariant Lagrangian one introduces a gauge variant functional of the effective fields,  $S_{\text{wzw}}$ , constructed so as to reproduce the change in the effective action [3,6,7,8,9]. Clearly, the crucial point here is that the functional is allowed to depend on the Goldstone field  $U$ ,  $S_{\text{wzw}} = S_{\text{wzw}}\{U, v, a, \dots\}$  – the very existence of anomalies is a manifestation of the statement that a local expression formed with the external fields alone cannot generate the change in (5). In this way, the anomalies of the underlying theory generate effective vertices for the Goldstone fields.

In the present case, the Wess-Zumino-Witten term does not involve the singlet axial field,  $S_{\text{wzw}} = S_{\text{wzw}}\{U, v, \hat{a}\}$ . It is given by the action of a suitable Lagrangian,  $S_{\text{wzw}} = \int d^4x \mathcal{L}_{\text{wzw}}$ , which can be written down in closed form:

$$\mathcal{L}_{\text{wzw}} = -\frac{N_c}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \left\{ \langle U^\dagger \hat{r}_\mu U \hat{l}_\nu - \hat{r}_\mu \hat{l}_\nu + i \Sigma_\mu (U^\dagger \hat{r}_\nu U + \hat{l}_\nu) \rangle \langle v_{\rho\sigma} \rangle \right. \\ \left. + \frac{2}{3} \langle \Sigma_\mu \Sigma_\nu \Sigma_\rho \rangle \langle v_\sigma \rangle \right\} , \quad (12)$$

where the quantities  $\hat{r}_\mu, \hat{l}_\mu$  and  $\Sigma_\mu$  stand for

$$\hat{r}_\mu = \hat{v}_\mu + \hat{a}_\mu , \quad \hat{l}_\mu = \hat{v}_\mu - \hat{a}_\mu , \quad \Sigma_\mu = U^\dagger \partial_\mu U .$$

A derivation of this result is presented in appendix B. To prove that this expression accounts for the anomalies of two-flavour QCD correctly, it suffices, however, to show that, under an infinitesimal chiral rotation of the fields  $U$ ,  $v$  and  $\hat{a}$ , it does generate the change given in eq. (5). That this is indeed the case may be verified by explicit calculation. It may also be checked that the expression given in eq. (12) respects parity and charge conjugation invariance. Naturally, the Wess-Zumino-Witten term is unique only up to gauge invariant contributions. Here, we have adapted the convention  $S_{\text{wzw}}\{\mathbf{1}, v, \hat{a}\} = 0$  (see appendix B).

As anticipated in the introduction, the Wess-Zumino-Witten Lagrangian in eq. (12) is considerably simpler than the standard expression for this term [7,8,9]. In particular, one notices that the term identified by Witten [7], the five-dimensional integral of the form

$$\int_{M_5} d^5x \epsilon^{ijklm} \langle \Sigma_i \Sigma_j \Sigma_k \Sigma_l \Sigma_m \rangle$$

is missing. The absence of this term is due to the fact that there is no anomaly in the correlation function of five axial vector currents. Its absence can also

be understood on the level of the effective theory: The four available fields,  $\pi^+, \pi^-, \pi^0$  and  $\theta$ , do not allow for a nonvanishing, totally antisymmetric fifth rank tensor. This implies that the five-point function is not only free from anomalies but, moreover, vanishes to the order considered. For two flavours, the anomalous interactions of the Goldstone fields are absent if the external fields are switched off. In fact, eq. (12) shows that they are proportional to the trace  $\langle v_\mu \rangle$  of the vector field, as was already discussed in section 2.

We add two remarks in connection with the  $U(1)_A$ -anomaly. First, we note that the Lagrangian (12) is independent of the singlet axial field  $\langle a_\mu \rangle$ . Accordingly, it represents a renormalization group invariant expression. Furthermore, this expression is independent of the vacuum angle  $\theta(x)$ . This follows from the fact that the Lagrangian in eq. (12) is invariant under chiral  $U(1)_A$ -rotations: The Lagrangian is the same independently of whether the matrix  $U$  is subject to the constraint  $\det U = e^{-i\theta}$  or  $\det U = 1$ .

## 5 Three-point functions

As an illustration, we give here the expressions for two examples of three-point functions determined by the unnatural parity part of the Lagrangian (11). As the first example we consider the correlation function of one axial and two vector currents,  $\langle 0 | T A^i V^k V^0 | 0 \rangle$ , which, to the order considered, is completely determined by the Wess-Zumino-Witten term. This correlation function determines the radiative decay of the neutral pion and contributes also to  $\pi^- \rightarrow e^- \bar{\nu}_e \gamma$ . The term relevant for the present calculation is the second one in the expression for the WZW-Lagrangian (12). The third contributes in an analogous manner, for instance, to the process  $\gamma \pi^0 \rightarrow \pi^+ \pi^-$ , while the first only matters when at least three gauge bosons are involved. A simple calculation leads to the result

$$\begin{aligned} i^2 \int dx dy e^{-ikx+ipy+iqz} \langle 0 | T A_\alpha^i(x) V_\mu^k(y) V_\nu^0(z) | 0 \rangle \\ = i \frac{N_c}{8\pi^2} \frac{\delta^{ik}}{M_i^2 - k^2} \epsilon_{\mu\nu\rho\sigma} k_\alpha p^\rho q^\sigma + O(p^3) , \end{aligned}$$

with  $k = p + q$ ,  $M_1 = M_2 = M_{\pi^+}$ ,  $M_3 = M_{\pi^0}$ . Upon contracting the result with  $k^\alpha$ , the anomalous divergence of this correlation function is easily identified (note that, for nonvanishing quark masses, the Ward identity involves the correlation function  $\langle 0 | T P^i V^k V^0 | 0 \rangle$  of the pseudoscalar current

$P^i = \bar{q}^i \gamma_5 \tau^i q$ ). The residue at  $k^2 = M_{\pi^0}^2$  determines the  $\pi^0 \rightarrow \gamma\gamma$  amplitude<sup>4</sup>. Integration over phase space leads to the well known prediction

$$\Gamma_{\pi^0 \rightarrow \gamma\gamma} = \frac{\alpha^2 N_c^2 M_{\pi^0}^3}{576 \pi^3 F_\pi^2} + O(m^{\frac{5}{2}}) ,$$

where  $\alpha$  denotes the fine structure constant.

In contrast to the above example, the correlation function  $\langle 0 | T A^i A^k A^0 | 0 \rangle$  does not receive a contribution from the WZW-term but is instead proportional to the effective coupling constant  $\tilde{\ell}_1$ ,

$$i^2 \int dx dy e^{ipx+iqy-ikz} \langle 0 | T A_\mu^i(x) A_\nu^k(y) A_\rho^0(z) | 0 \rangle = T_{\mu\nu\rho}^{ik}(p, q, k) + T_{\nu\mu\rho}^{ki}(q, p, k) ,$$

$$T_{\mu\nu\rho}^{ik}(p, q, p+q) = 8i \tilde{\ell}_1 \delta^{ik} \epsilon_{\nu\rho\alpha\beta} q^\alpha \left\{ \frac{p_\mu p^\beta}{M_i^2 - p^2} + g_\mu{}^\beta \right\} + O(p^3) .$$

In this case the divergences of the charged axial currents are anomaly free. In the present framework, this is true in general for the correlation functions involving the singlet axial current since these only receive contributions from the gauge invariant part of the Lagrangian – the WZW-term is independent of the field  $\langle a_\mu \rangle$ . In the effective Lagrangian, the anomalies of the singlet axial current manifest themselves in that the the field  $\langle a_\mu \rangle$  exclusively enters in combination with the vacuum angle, in the form  $D_\mu \theta = \partial_\mu \theta + 2 \langle a_\mu \rangle$ . One may check that in the above example the divergence of the singlet axial current is indeed proportional to correlation function  $\langle 0 | T A^i A^k \omega | 0 \rangle$ .

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<sup>4</sup> The electromagnetic current  $J_\mu^{\text{e.m.}}$  also receives a contribution from the 'heavy' quark flavours  $s, c, b, t$ ,

$$J_\mu^{\text{e.m.}} = J_\mu^\ell + J_\mu^h , \quad J_\mu^h = \frac{2}{3} \bar{c} \gamma_\mu c - \frac{1}{3} \bar{s} \gamma_\mu s + \frac{2}{3} \bar{t} \gamma_\mu t - \frac{1}{3} \bar{b} \gamma_\mu b .$$

To account for the contributions from the heavy quarks, we extend the list of external fields in eq. (2) so as to include a source field  $A^\mu$  for the current  $J_\mu^h$

$$\mathcal{L}_{\text{QCD}} \rightarrow \mathcal{L}_{\text{QCD}} - e A^\mu J_\mu^h .$$

Note that the current  $J_\mu^h$  is conserved. The extended framework thus exhibits an additional local  $U(1)_V$ -symmetry: The effective action is invariant under gauge transformations of  $A^\mu$ . Starting at order  $p^4$ , gauge symmetry permits corresponding additional terms in the effective theory:

$$\mathcal{L}_{\text{eff}} \rightarrow \mathcal{L}_{\text{eff}} + c_1 \langle v_{\mu\nu} \rangle F^{\mu\nu} + c_2 F_{\mu\nu} F^{\mu\nu} + O(p^6) ,$$

with  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ . The new terms, however, generate only contact contributions. To the order considered and what concerns matrix elements of pions, the electromagnetic current may therefore be identified with  $J_\mu^\ell = \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d$

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## A Anomalies

In the present appendix we derive the expression for the anomalies in two-flavour QCD given in eq. (5). For the following considerations it is convenient to introduce the differential forms

$$v = dx^\mu v_\mu, \quad a = dx^\mu a_\mu, \quad d = dx^\mu \partial_\mu.$$

The quantities  $dx^0, dx^1, dx^2, dx^3$  are treated as Grassmann variables. Their product yields the standard volume element,  $dx^\mu dx^\nu dx^\rho dx^\sigma = \epsilon^{\mu\nu\rho\sigma} d^4x$ .

The effective action of QCD is defined only up to contact terms formed with the external fields. It may be chosen such that it is invariant under the transformation generated by the vector charges. Accordingly, the change of the effective action under an infinitesimal chiral rotation (4) may be written in the form

$$\delta S_{\text{eff}}\{v, a, s, p, \theta\} = - \int \langle \beta \Omega(v, a) \rangle. \quad (13)$$

An explicit formula for  $\Omega(v, a)$  was first given by Bardeen [2] – it is valid for an arbitrary number of flavours  $N_f$ :

$$\begin{aligned} \Omega(v, a) &= \frac{N_c}{4\pi^2} \left\{ F_v F_v + \frac{1}{3} D_v a D_v a + \frac{i}{3} (F_v a^2 + 4a F_v a + a^2 F_v) + \frac{1}{3} a^4 \right\}, \\ F_v &= dv - iv^2, \quad D_v a = da - iva - iav, \end{aligned}$$

where  $v$  and  $a$  are  $N_f \times N_f$  matrices. Decomposing these into their traceless parts  $\hat{v}$ ,  $\hat{a}$  and a remainder,

$$v = \hat{v} + \frac{1}{N_f} \langle v \rangle, \quad a = \hat{a} + \frac{1}{N_f} \langle a \rangle,$$

$\Omega$  may be written as a sum of three terms,  $\Omega = \Omega_1 + \Omega_2 + \Omega_3$ ,

$$\begin{aligned} \Omega_1 &= \Omega(\hat{v}, \hat{a}) - \frac{1}{N_f} \langle \Omega(\hat{v}, \hat{a}) \rangle, \\ \Omega_2 &= \frac{N_c}{2N_f \pi^2} \{ \hat{F}_v + i\hat{a}^2 \} \langle dv \rangle, \\ \Omega_3 &= \frac{N_c}{6N_f \pi^2} \{ D_v \hat{a} \langle da \rangle - 2i(\hat{F}_v \hat{a} - \hat{a} \hat{F}_v) \langle a \rangle \} + \frac{1}{N_f} \langle \Omega(v, a) \rangle, \end{aligned} \quad (14)$$

with  $\hat{F}_v = d\hat{v} - i\hat{v}^2$ ,  $D_v\hat{a} = d\hat{a} - i\hat{v}\hat{a} - i\hat{a}\hat{v}$ . The significance of this decomposition is the following: The first term,  $\Omega_1$ , is the anomaly corresponding to the well known standard case where the singlet vector and axial vector currents are disregarded and the chiral transformations are restricted to the subspace  $SU(N_f)_R \times SU(N_f)_L$ . For  $N_f = 2$ , the second term,  $\Omega_2$ , is identical with the expression for the anomaly given in eq. (5) – its implications are discussed in detail in the present paper. The third term,  $\Omega_3$ , has the property that it represents the anomaly generated by a contact term  $P$ ,

$$P = \frac{N_c}{8N_f\pi^2} \left\{ \langle v(dv - \frac{2i}{3}v^2) \rangle d\theta + \langle aD_v a \rangle (\frac{4}{3}\langle a \rangle + d\theta) \right\} - \frac{N_c}{12N_f^2\pi^2} \langle a \rangle \langle da \rangle d\theta ,$$

with  $\delta \int P = -\langle \beta \Omega_3 \rangle$ . As stated above, we may redefine the effective action by removing this term,

$$S_{\text{WZW}} \rightarrow S_{\text{WZW}} - \int P .$$

With these conventions, the anomalies of  $N_f$ -flavour QCD are given by

$$\delta S_{\text{eff}}\{v, a, s, p, \theta\} = - \int \langle \hat{\beta} (\Omega_1 + \Omega_2) \rangle , \quad (15)$$

where we used the fact that both,  $\Omega_1$  and  $\Omega_2$ , are traceless. For two flavours,  $N_f = 2$ , the term  $\langle \hat{\beta} \Omega_1 \rangle$  reduces to zero and we are left with the result given in eq. (5).

Above, we have made use of the freedom in the definition of the effective action to bring the anomalies to a simple form. Correlation functions calculated with either version of the effective action differ only in contact contributions which are physically irrelevant. In one respect, however, our definition of the anomalies is conceptually superior to the original one: As pointed out in ref. [12], the form of the anomaly in eq. (13) fails to be renormalization group invariant, because it involves the singlet axial field  $\langle a_\mu \rangle$ . As a consequence of the anomalous dimension of the singlet current  $A_\mu^0$ , this field depends on the running scale of QCD – otherwise the perturbation generated by the term  $\langle a_\mu \rangle A_0^\mu$  would fail to be renormalization group invariant. While the right hand side of eq. (13) is in conflict with the scale independence of the theory, eq. (15) does represent a renormalization group invariant statement.

The requirement of renormalization group invariance only constrains that part of contact term  $P$  which involves the field  $\langle a_\mu \rangle$ . In addition, we have adapted here the convention that the change is independent of  $\langle \beta \rangle$  and  $\theta$ , as a consequence of which the anomalies of the singlet axial current become universal.

## B Construction of the WZW-term

In this appendix, we demonstrate how the Wess-Zumino-Witten term may be obtained from the explicit expression for the change in the effective action given in eqs. (5) and (14) [3,6,7,8,9]. We seek a functional  $S_{\text{WZW}}\{U, v, \hat{a}\}$  with the property that it reproduces the anomaly under an infinitesimal chiral transformation,

$$D(\alpha, \beta) S_{\text{WZW}}\{U, v, \hat{a}\} = - \int \langle \hat{\beta} \Omega_2 \rangle , \quad (16)$$

where  $D(\alpha, \beta)$  denotes the generator of infinitesimal chiral transformations [3,6]. The exponential of  $D(\alpha, \beta)$  generates finite chiral transformations:

$$\begin{aligned} e^{D(\alpha, \beta)} U e^{-D(\alpha, \beta)} &= V_{\text{R}} U V_{\text{L}}^\dagger , \\ e^{D(\alpha, \beta)} (v + a) e^{-D(\alpha, \beta)} &= V_{\text{R}} (v + a) V_{\text{R}}^\dagger + i V_{\text{R}} dV_{\text{R}}^\dagger , \\ e^{D(\alpha, \beta)} (v - a) e^{-D(\alpha, \beta)} &= V_{\text{L}} (v - a) V_{\text{L}}^\dagger + i V_{\text{L}} dV_{\text{L}}^\dagger , \end{aligned} \quad (17)$$

with  $V_{\text{R}} = e^{i(\alpha + \beta)}$ ,  $V_{\text{L}} = e^{i(\alpha - \beta)}$ . In order to render its solution unique, we proceed to equip the differential equation (16) with a boundary condition,

$$S_{\text{WZW}}\{\mathbf{1}, v, \hat{a}\} = 0 . \quad (18)$$

Note the consistency of this equation with the invariance with respect to vector transformations. (Eq. (16) shows that the Wess-Zumino-Witten functional is invariant under  $U(1)_{\text{A}}$ -rotations,  $S_{\text{WZW}}\{U e^{i\varphi}, v, \hat{a}\} = S_{\text{WZW}}\{U, v, \hat{a}\}$ . Here and in the following we may therefore disregard the associated ambiguities – the final result will be independent of these.) From the two preceding equations one infers

$$e^{D(\alpha, \beta)} S_{\text{WZW}}\{U, v, \hat{a}\} \Big|_{\alpha=\alpha_U, \beta=\beta_U} = 0$$

for a pair of matrices  $\{\alpha_U, \beta_U\}$  that satisfy the equation

$$e^{i(\alpha_U + \beta_U)} U e^{-i(\alpha_U - \beta_U)} = \mathbf{1} .$$

The solution to this equation is obviously not unique – the Wess-Zumino consistency conditions [3] guarantee, however, that the final result will be independent of the choice<sup>5</sup>. A particularly convenient choice is given by [9]

$$\alpha_U = -\beta_U , \quad U = e^{-2i\beta_U} , \quad \text{i.e.} \quad V_{\text{R}} = \mathbf{1} , \quad V_{\text{L}} = U . \quad (19)$$

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<sup>5</sup>In fact, at the cost of some algebraic complication, the subsequent steps may also be carried out without specifying a particular solution of the type (19)

Putting things together, we arrive at

$$\begin{aligned} S_{\text{WZW}}\{U, v, \hat{a}\} &= \sum_{n=1}^{\infty} \frac{1}{n!} \int \langle \hat{\beta} [D(\alpha, \beta)]^{n-1} \Omega_2 \rangle \Big|_{\alpha=\alpha_U, \beta=\beta_U} \\ &= \int_0^1 dt \int \langle \hat{\beta} e^{D(t\alpha, t\beta)} \Omega_2 \rangle \Big|_{\alpha=\alpha_U, \beta=\beta_U} \end{aligned} \quad (20)$$

where we used the linearity of  $D$  with respect to its arguments. When acting on  $\Omega_2$ , the operator  $e^D$  replaces the fields  $v$  and  $\hat{a}$  with their gauge transforms, cf. eq. (17):

$$e^{D(t\alpha, t\beta)} \Omega_2(v, \hat{a}) = \Omega_2(v_t, \hat{a}_t) .$$

Evaluating this at  $\alpha = \alpha_U$ ,  $\beta = \beta_U$  from eq. (19), we obtain

$$v_t + a_t = v + a , \quad v_t - a_t = U_t(v - a)U_t^\dagger + iU_t dU_t^\dagger , \quad U_t = e^{-2it\beta_U} .$$

Finally, using  $\partial_t U_t = -2i\beta_U U_t$ , the integration with respect to  $t$  in eq. (20) can be performed explicitly. The result is of the form

$$S_{\text{WZW}}\{U, v, \hat{a}\} = \int d^4x \mathcal{L}_{\text{WZW}} ,$$

and, for  $N_f = 2$ , the explicit expression for  $\mathcal{L}_{\text{WZW}}$  is the one in eq. (12).

The above derivation explicitly shows that, apart from the effective variables, the WZW-term exclusively involves the fields which occur also in the expression for the anomaly:  $v$  and  $\hat{a}$  in the present case. For this to be true, the subset of fields must be complete with respect to chiral transformations: Once again, the Wess-Zumino consistency conditions [3] ensure that this is the case.

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